2.1 Derivative Basics: Form I Assessment

Name: _____ Time: 60 minutes No calculator 100 points

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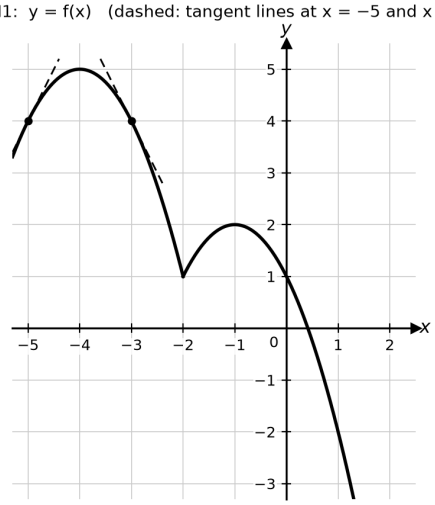
Answer exactly the question asked. Show work on every problem in Parts II–IV and Part VI.
If a derivative does not exist, write DNE. If a function value does not exist, write *undefined*.
Angles are in radians.

Part I. Graph. Questions 1(a)–1(f) refer to Figure I1. The dashed lines are tangent to the graph of f at $x = -5$ and $x = -3$.

Figure I1: y = f(x) (dashed: tangent lines at x = −5 and x = −3)



1. Use the figure to answer each part.

   (a) $f(-5)$ _____ (b) $f'(-3)$ _____

   (c) $f'(-1)$ _____ (d) $f'(-2)$ _____

   (e) Every open interval within $(-5, 2)$ on which $f'(x) < 0$: _____

   (f) Write $f'(-5)$, $f'(-3)$, and 0 in order from least to greatest: _____

   (2 pts each for (a)–(d), 3 pts each for (e)–(f); 14 pts)

Part II. Tangent lines and values. Show work.

2. Let $f(x) = x^3 + 4x^2 + 1$. Find an equation of the tangent line at $x = -1$. **(5 pts)**

3. Let $g(x) = 2x^2 - \dfrac{6}{x}$. Find the y-intercept of the line tangent to the graph of g at $x = 1$. **(5 pts)**

4. The line tangent to the graph of a function w at $x = -1$ has equation

$$y - 2 = 4(x + 1) - x - 3.$$

(a) Find $w'(-1)$. (b) Find $w(-1)$. **(3 + 2 pts)**

5. Let $v(x) = x^3 - x^2 - 4x$, so that $v'(x) = 3x^2 - 2x - 4$. Find $v(-2)$. **(2 pts)**

Part III. Definition of the derivative. Show work.

6. Let $f(x) = -\dfrac{6}{2x+3}$. Use the definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

to find $f'(x)$. An answer found with differentiation rules instead of the definition earns at most 2 points. **(8 pts)**

7. Let $g(x) = 2x^3 - 3x$. Use the definition

$$g'(c) = \lim_{x \to c} \frac{g(x) - g(c)}{x - c}$$

with $c = 1$ to find $g'(1)$. An answer found with differentiation rules instead of the definition earns at most 2 points. **(8 pts)**

8. Evaluate each limit. **(3 each, 6 pts)**

(a) $\lim_{x \to \pi/6} \dfrac{\cos x - \frac{\sqrt{3}}{2}}{x - \pi/6}$

(b) $\lim_{h \to 0} \dfrac{(1+h)^5 - 1}{h}$

Part IV. Differentiation rules. Show work.

9. Find each derivative. **(3 each, 6 pts)**

(a) $a(x) = \sqrt{2}\cos\left(\frac{\pi}{4}\right)$. Find $a'(x)$.

(b) Let $f(x) = 3x - 2x^2$, and let $m(x) = f(1)$ for every x. Find $m'(x)$.

10. Find $f'(x)$ for each function. **(3 each, 12 pts)**

(a) $f(x) = \dfrac{12}{\sqrt[3]{x}}$

(b) $f(x) = \dfrac{2x^3 + 8x}{4x}$

(c) $f(x) = x(x-1)(x+1) - 2\cos x$

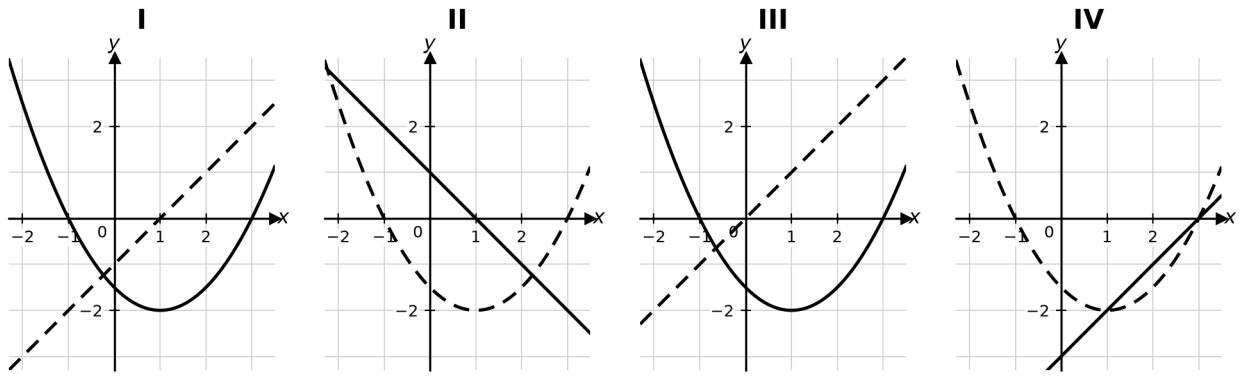
(d) $f(x) = 3x^4 - \dfrac{2}{\sqrt{x}}$

11. Let $f(x) = 5\cos x + 2x^2$. Find $f'\left(\frac{\pi}{2}\right)$. **(5 pts)**

12. Let $f(x) = x + \sqrt{2}\sin x$. Find every x in the interval $(\pi, 2\pi)$ at which the graph of f has a horizontal tangent line. **(6 pts)**

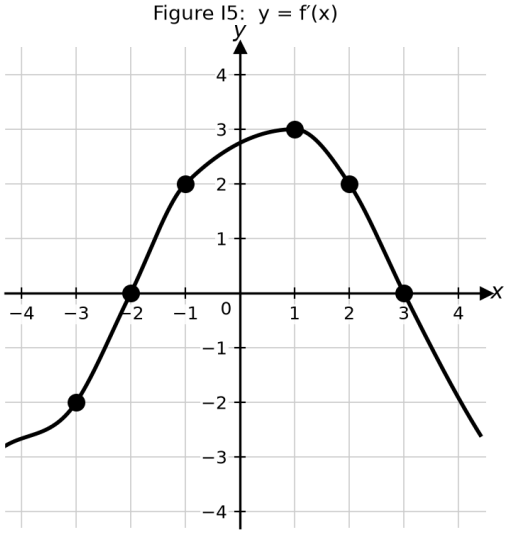
Part V. Graph of a derivative.

13. Each graph below shows two curves on the same axes. In exactly one of the graphs, one curve is the derivative of the other. Circle the Roman numeral of that graph. **(6 pts)**



Part VI. Rules with a graph.

14. Figure I5 shows the graph of f', the *derivative* of a function f. The marked points are on the graph of f'. **(4 each, 12 pts)**



(a) Let $h(x) = 2f(x) - x^2$. Find $h'(-1)$. _____

(b) Every x in $(-4, 4)$ at which the graph of f has a horizontal tangent line _____

(c) The x in $[-4, 4]$ at which the line tangent to the graph of f has the greatest slope _____